Neumann and Nash: a Review of " A Beautiful Mind"

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The Nobel Prize for Economics in 1994 was presented to J. Harsanyi, R. Selten and J. Nash. The decision was taken in order to celebrate the fiftieth anniversary of the publication of *Theory of Game and Economic Behavior* by John von Neumann and Oskar Morgenstern. Harsanyi is seventeen years younger than Neumann and both of them are alumni of the Lutheran Gymnasium of Budapest. There were other Hungarians at Princeton around 1950 when Nash was a Ph.D. student there; John Kemeny was Albert Einstein's assistant at the Institute of Advanced Study. Einstein was not a master of modern mathematics and frequently required the help of the young mathematician. Eugen Wigner, designer of the first ever reactor was Professor of Physics. Wigner also graduated from the Lutheran Gymnasium a year earlier than Neumann. Peter Lax, who graduated from the Minta Gymnasium in Budapest, was a Ph.D. student with whom Nash had a good relationship and sought advice and consultation from [(1)].

The Film: *A Beautiful Mind*

Compared to the lives of Neumann and Erdős, who both traveled to the four corners of the globe as key historical events unfolded around them, Nash's life is limited to rather narrow and confined boundaries. This hardly makes him the appropriate subject for a film. Despite this, Nash's life is not so uneventful. Without warning in 1959 he is diagnosed as a schizophrenic and after 25 years rehabilitation he succeeds in returning to teaching. A few years later he learns that somehow he has won the Nobel Prize for something he wrote for his PhD thesis. This dramatic story was made into a film by virtue of it characterizing a typical American success story, with a side plot focusing on his wife, who devotes her life to the unwavering care of her sick husband.

The film begins with Nash entering the Graduate School of Princeton University in 1948. His fellow students see Nash as an eccentric who shows an increasingly arrogant streak with regard to his talent. They also notice that Nash has the curious tendency of making childish jokes at the expense of his friends. Although he struggles a great deal with writing his thesis, he eventually completes a mathematical model entitled "Bargaining Problem"(1950). This just so happens to win him Nobel Prize forty-four years later [(2)].

Thanks to his thesis Nash gets a lecturing post at the University and is also asked to put his mathematical ability into code breaking for a military institution. Then the plot thickens: Nash becomes totally obsessed with codes, and tries to read every single possible code, even those he spots in magazines or newspapers. At the same time he gets drawn into spying and counter intelligence. Ultimately, he loses his senses, unable to distinguish reality from what is going on in his mind.

After the onset of his schizophrenia in 1959, he is forced to stand down from his university post and his wife commits him to a mental hospital for treatment. Nash undergoes electric shock treatment and after the initial bout cannot face the prospect of more. His wife becomes determined to take care of him by herself at the couple's home. After more than 25 years of her devoted care his hallucinations subside and with his schizophrenia under control, Nash is finally able to return to university teaching in the

mid 1980's. Then one day, a few years later, the unthinkable happens as Nash learns that he has won the Nobel Prize. Now it is Nash who receives the private ceremony of professors in the faculty club, where once he sat observering others during his days as a Ph.D. student.

A Short Opinion about the Film

The borderline between the real and imagined code cracking carried out by Nash is not clear from the film. The film intentionally mixes the real and imaginary worlds of Nash, allowing the viewer a certain amount of insight into the causes of his disease. The film may suggest that it is Nash's spying activity that destroys him mentally, since the film is set during the 1950's when MaCarthyism was at its height. However, the film does not portray the social conflict between scientists and the military that was going on at the time, even though the date is displayed throughout the film.

Furthermore, in spite of the film showing the year the scenes are set, it neglects the most important events of the day, such as the social friction and conflict in 1950's society. This for me is the fundamental flaw of the film, just as is the case with a large number of films from the US.

The Original Body of Work: *The Ugly Mind*

After reading the biography, also entitled "A Beautiful Mind", that the film is based on, it is apparent that the film and the biography [(3)] are entirely different to one another. It goes without saying that it can be argued that the two are entirely independent creative works and nobody can be blamed for the differences. Oddly enough, I arrived at the title "The Ugly Mind" after reading just half of the original work. This for me accurately depicts the unpleasant nature of the behavior of the young Nash.

Unlike Neumann, whose genius was apparent from his childhood, Nash's mathematical ability came to the fore in his latter school years. Once he became aware of his ability, his ambition grew and he harvested a strong desire to reach top status as a mathematician, so as to land a tenure in Princeton or Harvard University. He was standoffish and socially inadequate, his personality a mix of a child's tinged with an arrogant streak. Due to his peculiar traits, he was unable to claim the teaching post in Princeton University, but settled for MIT whose academic ranking was lower that Princeton at that time. This was his first setback in life.

The RAND Corporation, where Nash was a contract researcher engaged in the study of "game theory", was by no means a sanctuary from the MaCarthy purge. At that time homosexuality was considered a fatal defect in anybody defending state secrets, and consequently homosexual scientists were deprived clearance to the top secrets. It is for this reason that Nash was arrested by an undercover policeman in a public lavatory situated near the RAND Corporation in 1954, and not because of possible counter espionage. He was deprived of top-level clearance and was discharged from RAND. Even under these circumstances, however, it was always his prime interest to avoid compulsory military service. Although a few mathematicians in RAND and Princeton were also arrested for the same reason and committed suicide, Nash didn't pay much his attention to it, and refused to take it that seriously. In this sense he lacked the understanding of the social climate of his time. Contrary to the character portrayed in the film, the real Nash had a son with his girlfriend, but he didn't support her in bringing up

his child. Therefore, his first son was put into a children's home and moved about from family to family. It was highly selfish behavior of Nash to leave his child in this way.

As far as can be deduced from the biography, the direct cause of his schizophrenia, appears to stem from his failure to be selected as one of the candidates for the Fields Medal and Bôcher Prize in 1958. In this year Nash sent the same paper to two mathematical journals, *Acta Mathematica* (the Swedish Mathematics journal) and *American Journal of Mathematics*, since a prize winning article had to be published. The editor of *Acta Mathematica*, Carleson, saw through every arrangement for clearing Nash's article and informed him that it would be published. However, Nash withdrew his article from the journal, when he became aware that the Bôcher Prize is given to articles published only in the USA. In this way he violated the rule of professional ethics. This story shows how he grew impatient with building his reputation as a mathematician to secure a tenure in a university of top reputation, such as Harvard and Princeton. Dejected at the prospect of a four-year wait until the next prizes were to be awarded, Nash spent Christmas and New Year highly frustrated, breaking down entirely in spring.

The phrase of "A Beautiful Mind" was given by Shapley to Nash after receiving the Nobel Prize. Shapley was a rival researcher in game theory and he praised Nash for overcoming his arrogance and childishness of his youth by conquering his disease. Nevertheless, the phrase should not be used to describe Nash, but his wife and his friends who helped and supported Nash by giving him the opportunity to keep studying at a university, providing material assistance. The generous and considerate treatment for the handicapped in US universities is in itself impressive.

Doubts about the Nobel Prize in Economics

The biography features one scene of a meeting between Nash and Neumann, who at the time was already a prominent figure in the academic world and exerted a strong influence on the US military forces. The author, Sylvia Nasar, notes that after listening to Nash's explanation Neumann interrupted him and said, "That's trivial, you know. That's just a fixed point theorem." This dealt a crushing blow to Nash, who felt undermined by Neumann. It was Nash's first and only meeting with him. Nasar put Neumann's behavior down to defending himself against a young rival. This is too simple an explanation and does not reflect what actually happened, since Neumann had already departed from researching game theory. Neumann had handed the opportunity to develop the theory over to younger researchers at the RAND Corporation, where Nash also became involved with it. The author's foreword to the third edition of *Theory of Game and Economic Behavior* (January, 1953), in which Neumann lists several studies as new developments to game theory, includes Nash's contribution to the theory of the n-person game.

However, Neumann's point may be very interesting from a mathematician's standpoint [(4)]. As Nasar describes in detail, the decisions that led to the award of the Nobel Prize of Economics in 1994 was both many fold and troublesome, not only because of Nash's disease, but also due to the scientific evaluation of game theory itself. In the formal meeting of the Royal Swedish Academy of Sciences, the three candidates got the minimum approval that was necessary to receive the prize. After the tumultuous event, the economic committee members were discharged and modifications were made to the economics prize. This saw the field widened from pure economics to social sciences. The essence of the problem was that three candidates articles were exclusively of a

mathematical background. On one hand, the level of mathematics used in game theory is nothing out of the ordinary, but it is ordinal and could be referred to as elementary from a pure mathematics perspective. The three became candidates for the Nobel Prize only because of their application of mathematical theorems to economics. On the other hand, the question of how game theory has been applicable to real economy comes to light. Interestingly enough, in answer to this question, no persuasive contributions and real examples have ever materialized. Only mathematical models have existed. Here it is worthwhile examining the contemporary history of mathematical economics, which in fact actually started with Neumann.

Neumann as the Pioneer of Contemporary Mathematical Economics

It is very interesting to establish how Neumann began to study economic theory. According to Norman Macrea, Neumann got together with Nicholas Kaldor (Kárdor Miklós), an economist in Budapest, when the pair returned home during vacations. Although their ideological viewpoints are strikingly different, they were nevertheless able to establish a strong friendship. Through discussions with Kaldor, Neumann learned about main subjects in economics, including the equilibrium theory of L. Walrus, which called for more precise mathematical treatment at the time.

Thus, as he usually did, Neumann studied the fundamental points of the problems and rapidly came up with mathematical models that covered two areas: one for game theory and the other for equilibrium theory. A paper of the former was published in 1928 [(5)] which formulized the Mini-Max theorem, and the latter was initially lectured in Princeton, 1932 and then published in 1936 [(6)] at the request of K. Menger. Menger organized a mathematical colloquium in Vienna (the Vienna Colloquium) in which several prominent mathematicians and economists including A. Wald, K. Schlesinger and O. Morgenstern participated. In the equilibrium model Neumann firstly described a system of inequality for economic models and used topological methods to prove the existence of an equilibrium point. No one had ever used the fixed-point theorem in an economic model, but Neumann was the first to show how the theorem could be used in a mathematical economics model. Actually Neumann extended Brouwers's Theorem of Fixed Point for the model, and then S. Kakutani refined the extension as "Kakutani's Theorem of Fixed Point" when he visited Princeton in 1941 as a guest researcher. From mathematical methods Nash used both Brauwer's and Kakkutani's theorems to prove his own equilibrium. Hence, without hesitation Neumann pointed out that there were no new mathematical ideas in his model.

Inequality, convex set, saddle point and fixed point: these all are the mathematical concepts that Neumann was the first to introduce into mathematical economics. He did so in order to rewrite old style economic models written in the classical analysis of the 19th century, to establish economic models equipped with the topological methods belonging to the 20th century. In the same way, *Theory of Game and Economic Behavior* was not just intended for game theory itself, but also to provide a new foundation to microeconomics. Thus, the impact of Neumann's work in mathematical economics was deep and revolutionary. It cannot therefore be compared to Nash's work, with his brief paper merely aimed at extending the game theory model.

The starting point of mathematical economics after World War Two was derived from studying Neumann's work. Nobel Prize winning economists, such as T. Koopmans, K.

Arrow, G. Debreu, P. Samuelson and R. Solow all studied and taught Neumann's books and papers as they established their respective careers. The three economists who received the Nobel Prize in 1994 were students of Neumann in the sense that they extended the model of game theory that was initially formularized mathematically by Neumann.

Nasar exaggerates Nash's work and declares that Nash's findings were revolutionary in their effects on economic history [(7)]. Her evaluation is too naive. The extension of a model from two persons to n-persons, from complete informational to incomplete informational, from cooperative to non-cooperative, and so on are nothing more than a study in the course of model generalization. Moreover, we should be careful that the generalization of a model in itself does not mean a widening of the model's application. On the contrary, models are losing their relevance to reality, as they are more and more generalized in the sense of mathematics. Ironically, limited models with concrete conditions are the most relevant and applicable to the real world, such as the two-person game.

It would be interesting to put a question to Neumann, if he were alive, on how he evaluates the present Nobel Prize of Economics. I think he would not accept the prize unless he received it as the first ever winner. "Economics is not my field, science is!" he might say. Although Neumann contributed to many fields of science, he himself listed only two things as his prominent work: one is the mathematical foundation of quantum physics and the other is the exact proof of quasi-ergodic theorem. All else is of secondary or third importance for him.

(1) See G. Marx, *The Voice of Martians*, Akademiai kiado, Budapest,2001.
(2) Papers are found in H. Kuhn and S. Nasar(eds.), *The Essential John Nash*, Princeton University Press, 2002.
(3) Sylvia Nasar, *A Beautiful Mind*, Simon & Schuster, 1998.
(4) J. Milnor's paper is highly useful for understanding viewpoint of mathematician. John Milor, John Nash and "A Beautiful Mind", *Notices of the American Mathematical Society*, Volume 45, Number 10 (November 1998).
(5) John von Neumann, Zur Theorie der Gesellschaftsspiele, *Mathematische Annalen*, 100(1928).
(6) John von Neumann, Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunksatzes, *Ergebnisse eines Mathematischehn Kolloquiums,* Heft 8(1935-36).
(7) The same shortsighted view can be read in Roger B. Myerson, Nash Equilibrium and the History of Economic History, *Journal of Economics Literature*, Sept., 1999.